Optimal Spacecraft Attitude Control Using Collocation and Nonlinear Programming

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Introduction

IN recent years, NASA has been able to retrieve disabled satellites from orbit and has even repaired one satellite, the Solar Maximum satellite, while in orbit. However, there are many satellites in orbits too high for the shuttle to reach. If such a satellite fails to deploy properly or malfunctions after some use is made of it, there is currently no way of retrieving it. A remotely operated satellite [or orbital maneuvering vehicle (OMV)] is needed for retrieval of such satellites. A prerequisite for retrieval is the detumbling of the satellite.

Optimal open-loop controls for detumbling of a satellite were developed by Conway and Widhalm.[1] A nonlinear feedback control for the same problem was developed by Widhalm and Conway.[2] Webber[3] improved the nonlinear feedback controller of Ref. 2 and was able to make the cost comparable to that of the open-loop control formulation. Here, a new method, direct collocation with nonlinear programming (DCNLP), is employed to find the optimal open-loop control histories for detumbling a disabled satellite. The satellite system is assumed to have docked in such a way that the tumbling motion of the disabled satellite is not disturbed. The controls are torques and forces applied to the docking arm and joint and torques applied about the body axes of the OMV. The results of Ref. 1 have been reproduced, verifying the solution, but the method used for this work is simpler to implement and more robust. Solutions have been obtained for cases in which various constraints are placed on the controls and in which the number of controls is reduced or increased from that considered in Ref. 1.

Statement of the Problem

The basic problem is essentially as described by previous researchers.[1-3] The axisymmetric disabled satellite is assumed to be in a state of steady spin and precession, and the docking of the OMV with the disabled satellite is accomplished so that there are no coupling effects between the two bodies. The configuration of the two-body system just after docking is shown in Fig. 1. A basis $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ is fixed in the OMV and has its origin at the center of mass of the OMV, E^*. Similarly, a basis $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ is fixed in the disabled satellite and has its origin at the center of mass of the disabled satellite N^*. The two bodies are connected through a docking arm as shown. The arm joint can translate along the $\hat{e}_2$ axis and the arm can rotate at the joint about the $\hat{e}_1$ axis by an angle γ_1. Finally, the disabled satellite can spin at a rate $\dot{\gamma}_2$ about the docking arm, which is aligned with the $\hat{n}_3$ axis.

The desired final state of the two-body system is stable spin with the relative motion between the two bodies eliminated and the docking arm angle and joint position driven to zero. Thus, the disabled satellite spin rate $\dot{\gamma}_2$, coning angle γ_1, and joint position J_r are all to become zero. The time specified for the maneuver is chosen arbitrarily to be 300 s. Additionally, the OMV body rates about the $\hat{e}_1$ and $\hat{e}_2$ axes are driven to zero so that the two-body system will be spinning about the $\hat{e}_3$ axis at the final time. This will enable the two-body system to be locked together in preparation for transfer back to low Earth orbit.

Received July 1, 1990; revision received July 17, 1991; accepted for publication July 25, 1991.

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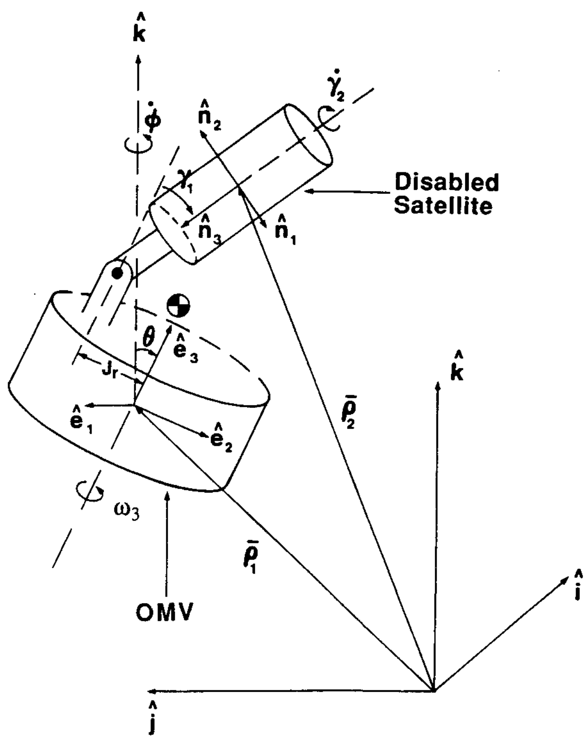

Fig. 1 OMV/disabled satellite system.

The equations of motion used in this work were derived by Webber[3] employing Kane's method.[4] To isolate the quantities of interest algebraically from these equations, the program MACSYMA[5] was used. These differential equations (DEs) govern the rotational and translational velocities of the OMV body, the docking arm angular rate and position, and the disabled satellite spin rate and position. The translation of the arm's joint is specified in some cases and allowed to be a control variable in others, in which case the trivial differential equations for variations of the joint velocity and position are added to the system DEs.

Optimal Control Using Nonlinear Programming

The cost function (for integral-square control)

$$\mathrm{Cost}(\bar{u}, t) = \frac{1}{2}\int_{t_0}^{t_f} \bar{u}^T B \bar{u}\, \mathrm{d}t \tag{1}$$

is used where $\bar{u}$ is a vector of length m, which represents the controls. The weighting matrix B is $m \times m$ and diagonal; t_0 and t_f are the initial and final times. The vector $\bar{u}$ is ordinarily the torque vector

$$\bar{T}_E = T_1\hat{e}_1 + T_2\hat{e}_2 + T_3\hat{e}_3 - TG_1\hat{e}_1 - TG_2\hat{n}_3 \tag{2}$$

where T_1, T_2, and T_3 are the external torques applied to the OMV, and TG_1 and TG_2 are internal torques applied to the disabled satellite by the OMV arm.

In Ref. 1, the optimal control problem was formulated using the fundamental optimal control theory.[6] Previous experience of the authors in applying the DCNLP method led us to believe that it would be more robust and efficient than a two point boundary value problem (TPBVP) solver. In this method,[7] the solution (or time history) of a first-order differential equation is discretized using time segments of equal length T. The state within each segment is approximated by a cubic polynomial in time, whereas the control is approximated by a linear function

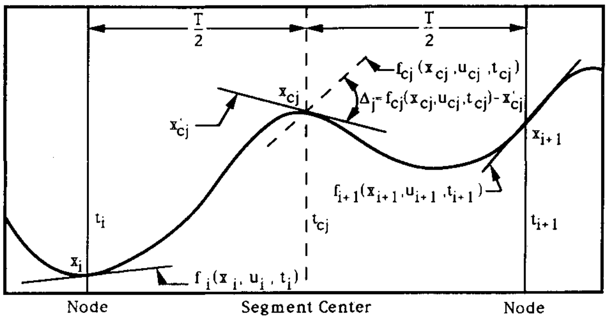

Fig. 2 Illustration of the collocation defect.

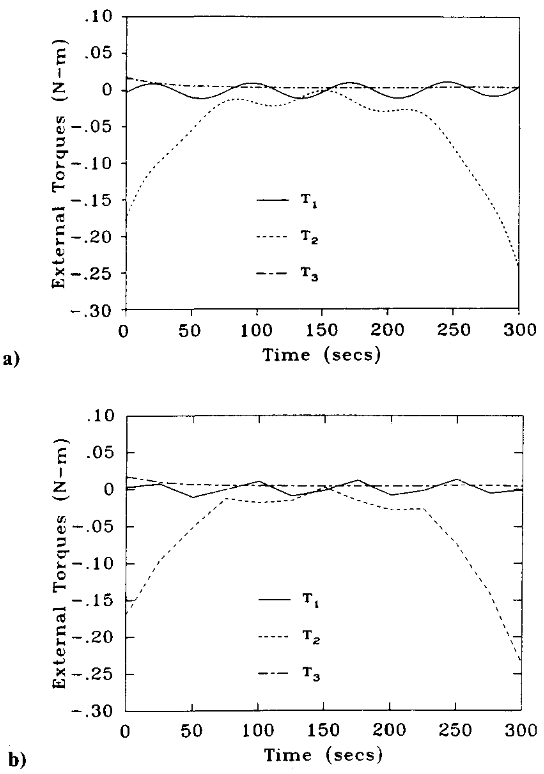

Fig. 3 External torque histories: a) using the TPBVP solver; b) using the DCNLP method.

in time. Figure 2 shows how the cubic polynomial for the jth segment may be determined using the values of the state at the nodes (or boundaries) of the segment x_i and x_{i+1}, and the slopes of the polynomial may be determined by evaluating the differential equation at the boundaries of the segment, i.e., $f(x_i, u_i, t_i)$ and $f(x_{i+1}, u_{i+1}, t_{i+1})$. The value of the state x_{cj} at the segment center t_{cj} is then determined using this polynomial. The value of the control u_{cj} at the segment center is determined using linear interpolation. These values at the segment center are used to determine the value of the differential equation $f(x_{cj}, u_{cj}, t_{cj})$ at the segment center, which is then compared with the first derivative of the cubic polynomial x'_{cj} at the segment center. The difference between these two quantities is termed a defect and may be seen to be the difference in the slope of the cubic and the value of the corresponding differential equation evaluated at the center of the segment, as shown in Fig. 2. For the problem of detumbling a disabled satellite, we have a set of n first-order differential equations that must be solved for the given initial conditions and desired final conditions. This results in a vector of n defects $\bar{\Delta}_j$,

$$\bar{\Delta}_j = \bar{f}(\bar{x}_{cj}, \bar{u}_{cj}, t_{cj}) - \bar{x}'_{cj} \tag{3}$$

for the jth segment.

Table 1 Spacecraft mass properties

Spacecraft	Mass, kg	I_{11}, kg-m^2	I_{22}, kg-m^2	I_{33}, kg-m^2
Orbital maneuvering vehicle	4500	6400	6400	11,800
Disabled satellite	1000	1000	1000	1100

Table 2 State boundary conditions

State	t_0	t_f
ω_1	0.0	0.0
ω_2	0.0	0.0
ω_3	-0.102 s^{-1}	free
$\dot{\gamma}_1$	0.0	0.0
$\dot{\gamma}_2$	-0.00871 s^{-1} [a]	0.0
γ_1	0.349 rad	0.0

[a] Approximate value.

A nonlinear programming routine (E04VCF[8]) is used to determine the states $\bar{x}_i$ and controls $\bar{u}_i$ that drive the defects to zero, or nearly to zero, simultaneously satisfying any (usually linear) system boundary conditions and minimizing the cost function $\text{Cost}(\bar{x}, \bar{u}, t)$. When the vector of defects has been driven to zero, an approximate solution of the system differential equations has been obtained. The problem may be stated as the following:

Minimize:

$$\text{Cost}(\bar{x}, \bar{u}, t)$$

Subject to:

$$\bar{\Delta}_j = \bar{f}(\bar{x}_{cj}, \bar{u}_{cj}, t_{cj}) - \bar{x}'_{cj} = 0, \qquad \text{for } j = 1, N_{\text{seg}}$$

$$\bar{x}_1 = \bar{x}(t_0)$$

for the initial conditions,

$$\bar{x}_L = \bar{x}(t_f)$$

for those states that have specified final conditions where N_{seg} is the number of segments, and L is the number of segment boundaries (or nodes) and is equal to $N_{\text{seg}} + 1$. The nonlinear programming routine E04VCF[8] is then used to vary the values of the states $\bar{x}_i$ and controls $\bar{u}_i$ at the nodes to determine a solution of the problem. This program is a comprehensive nonlinear programming (NLP) problem solver that minimizes an arbitrary, smooth function, $\text{Cost}(\bar{x}, \bar{u}, t)$, subject to any combination of simple bounds on the independent variables $\bar{x}_i$ and $\bar{u}_i$, linear constraints, and nonlinear, smooth constraints. Only simple bounds on the independent state variables at the initial and final times $\bar{x}_1 = \bar{x}(t_0)$ and $\bar{x}_L = \bar{x}(t_f)$, respectively, and nonlinear constraints $\bar{\Delta}_j = 0$, are used. No linear constraints are present in this problem. An initial guess of the independent variables must be given to E04VCF. The NLP routine also requires the gradient of the cost function and the Jacobian of the defects with respect to the independent variables. These values are computed by using the numerical differentiation technique of forward differencing with a step size equal to the square root of the relative machine precision. All computations were done on a Cray X/MP.

Examples

Many cases were successfully solved using this method. For all cases, the spacecraft mass properties and initial and final conditions are as given in Tables 1 and 2. These are the same as used in Ref. 1. The initial value of the disabled satellite spin rate $\dot{\gamma}_2$ and coning angle γ_1 are chosen to produce steady spin and precession, as described by Greenwood.[9] In case 1, the joint was considered to move at a constant velocity from the initial position to the $\hat{e}_3$ axis in 300 s. The cost function matrix B is the 5×5 identity matrix. The initial value of the joint position is approximately 0.598 m, h_e is 0.62 m, and L_1 is 1.75 m.

The NLP routine progressed smoothly to the optimal solution even though the initial guess of the state and control histories gave no information about the optimal solution. The control histories and optimal cost varied with the number of segments used. The use of 6, 12, 18, and 24 segments resulted in optimal costs of 2.941, 2.301, 2.317, and 2.320, respectively. When 12 or more segments are used, the cost compares well to the previous result of 2.335.[1] The results presented here are for the 12-segment formulation that required 30 s of execution time.

The solutions for the external torques T_1, T_2, and T_3, shown in Figs. 3, compare very well with those of Ref. 1. Webber[3] showed that changes in the joint acceleration profile may significantly affect the control histories and optimal cost. In the previous case, the motion of the joint is prescribed. When joint acceleration $\ddot{J}_r$ is instead included as an additional control variable and appropriately weighted in the cost function ($B_{66} = 2 \cdot 10^6$, leaving the weights on the other controls unchanged), the cost is significantly reduced to 0.726. As a third case, since thrusters normally have a fixed output, we have also examined the case in which the external torque controls are restricted to be constant throughout the detumbling process. Then, the optimal external torque magnitudes are found. For this case, the cost is 0.767, slightly larger, as expected, than the case in which the external torques are unconstrained. Finally, since the spin rate of the OMV, ω_3, is unspecified at the final time, and the attitude of the OMV is unspecified at the final time, we have solved a case in which the external controls are eliminated. This forces the internal controls to perform the entire work of detumbling the disabled satellite but would eliminate the need for fuel to be used during the detumbling process. The optimal cost for this case is 0.828. Space does not permit us to show the motion and control time histories for all of these cases, but they are available.[10] One observation that is true for all of the cases described is that the detumbling process is very benign; no large angular accelerations are produced. In addition, the required external torque magnitudes are very modest, on the order of 0.01-0.001 N-m.

Conclusions

The method of direct collocation and nonlinear programming presented here works well when applied to the optimal control problem of satellite attitude control. The formulation is straightforward and produces good results in a relatively small amount of time on a Cray X/MP with no a priori information about the optimal solution. The addition of the joint acceleration to the controls significantly reduces the control magnitudes and optimal cost. The restrictions of constant external torques or zero external torque have an insignificant effect on the state and internal control histories and result in only a small increase in the optimal cost. In all cases, the torques and accelerations are modest and the optimal cost is very modest.

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Equivalence of Kane's, Gibbs-Appell's, and Lagrange's Equations

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Introduction

In a recent Note,[1] Kane's generalized forces and equations were derived from a first principle—the work-energy form of Newton's second law. Lagrange's equations were also derived from this form; although it differs conceptually from the usual virtual work (d'Alembert principle) derivations, the steps are similar. These parallel derivations showed the commonality of Kane's and Lagrange's equations and the automatic occurrence of the virtual work terms, which are identified as geometric work. In this Note, the Gibbs-Appell equations are derived via a modification of the derivation of Kane's equations, and the equivalence of these three forms is derived for both the holonomic and nonholonomic cases.

The common feature of these equations is the transformation to generalized coordinates such that system constraint forces can be eliminated—a great convenience. In the usual derivations, this is attributed to the properties of virtual displacements[5-13] or other displacements that are chosen properly.[2,11,14] In all cases, explicit time-varying (rheonomic) kinematrical terms are discarded, often without comment, or with explanations that are convoluted and unenlightening.[2,5-13,15,16] The present derivations entail no such assumptions or explanations in establishing these results.

Derivations

The derivations are for a system of p constant-mass particles. Extension to rigid bodies is straightforward. Newton's second law for mass m_i, located at $\boldsymbol{r}^i$ in an inertial reference frame is

$$\boldsymbol{F}_i = m_i \ddot{\boldsymbol{r}}^i = m_i \boldsymbol{a}^i, \qquad i = 1, p \tag{1}$$

Here, $\boldsymbol{F}_i$ is the sum of all forces acting on the particle: the known applied and external forces and the constraint forces. The total work of all forces on the p particles undergoing some possible displacements $\mathrm{d}\boldsymbol{r}^i$, $i = 1, p$, is

$$\mathrm{d}W = \sum_{i=1}^{p} \boldsymbol{F}_i \cdot \mathrm{d}\boldsymbol{r}^i = \sum_{i=i}^{p} m_i \ddot{\boldsymbol{r}}^i \cdot \mathrm{d}\boldsymbol{r}^i = \sum_{i=1}^{p} m_i \boldsymbol{a}^i \cdot \mathrm{d}\boldsymbol{r}^i \tag{2}$$

Received April 29, 1991; revision received Aug. 23, 1991; accepted for publication Sept. 3, 1991.

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